

TOPICS, TERMS and FORMULAS, ASTR 503

General text shorthand...

$a*b$ means a times b (the product of two terms multiplied together). Don't use "x" which can be a variable.

a/b means a divided by b

$a**b$ means a raised to the b power; $a**2$ is a-squared, etc.

aEb means a times ten to the b power. Thus $4.3E4$ is 43,000.

Note: this shows the number of significant figures at the same time!

Sqrt (a) means the square root of a. Same as $a**.5 = \sqrt{a}$

Sin (a), Cos (a), Tan (a) means sine, cosine, and tangent of the Angle a.

IF NO UNITS, assume the angle is measured in RADIANS (see below).

that = greek theta = θ (usually a measure of angle)

lam = greek lambda = λ (latitude, or wavelength)

alph = greek alpha = α (usually a measure of angle)

~ means "approximately equal to"

deg = degrees (usual units, 360 deg = a circle)

pi = π = you know this one. Approximately 3.14159.... my favorite number $\sim \sqrt{10}$

R = the radius of a circle

Re = the radius of the Earth (generally the "e" is an UPPERCASE subscript) = 6378 km.

qv = means "quid videre" which means "go look it up" (it's defined elsewhere).

In the html version this will be a hot link.

Scientific notation: write as $n.f\text{ff} \times 10^e$ where n is a number 1-9, .fff is the fraction (with the number of digits indicate how accurately we know the number), and e is the exponent (power of ten).

Thus, 1,000,000 = 1×10^6 if you only know one significant digit

1,001,000 = 1.001×10^6 since presumably that extra 1 means something, so four significant digits

PREFIXES IN SI (no prefix: plain unit, e.g. m = meters)

deci = d =	0.1 = 10^{-1}	deka = da =	10 = 10^1
centi = c =	0.01 = 10^{-2}	hecto = h =	100 = 10^2
milli = m =	0.001 = 10^{-3}	kilo = k =	1000. = 10^3
micro = mu =	$E-6 = 10^{-6}$	Mega = M =	$E6 = 10^6$
nano = n =	$E-9 = 10^{-9}$	Giga = G =	$E9 = 10^9$
pico = p =	$E-12 = 10^{-12}$	Tera = T =	$E12 = 10^{12}$
femto = f =	$E-15 = 10^{-15}$	Peta = P =	$E15 = 10^{15}$
atto = a =	$E-18 = 10^{-18}$	Exa = E =	$E18 = 10^{18}$
zepto = z =	$E-21 = 10^{-21}$	Zetta = Z =	$E21 = 10^{21}$
yocto = y =	$E-24 = 10^{-24}$	Yotta = Y =	$E24 = 10^{24}$

constants you should know: speed of light $3 E5 \text{ km/s} = 3 E8 \text{ m/s}$ (186,000 miles per second)

Astronomical unit = AU = $1.5 E8 \text{ km}$ (93 million miles) = distance from Earth to Sun

Light year = distance light travels in a year = $c \text{ km/s} * \pi \times 10^7 \text{ s/yr} = 9.46 \times 10^{12} \text{ km}$

(seconds in a year = $365.25 \times 24 \times 60 \times 60 = 3.156 \times 10^7$)

1 parsec / 1 AU = 1 radian / 1 arc sec = $57.3 \text{ deg/rad} * 60 * 60 = 2.06 \times 10^5$

So, 1 parsec = $2.06 \times 10^5 * 1.5 \times 10^8 \text{ km} = 3.09 \times 10^{13} \text{ km} = 3.27 \text{ LY}$

Luminosity of the Sun = $L_{\odot} = 3.8 E 26 \text{ W}$

Absolute magnitude of the Sun = $M_{\odot} = 4.83$ Apparent magnitude of the Sun = $m_{\odot} = -26.74$

Mass of Sun = $2 E 30 \text{ kg}$

Introduction

1. Ask your students: what's in the sky? (answers might include stars, birds, clouds, the Sun)
Which things belong to earth and which are outside the earth?

Birds: you know they are of earth because they might land beside you

Clouds: you know they are of earth because they are sometimes below you (when you're on a mountaintop, or in a plane)

What about the Sun? How do we know it's not in our atmosphere? The Moon? Stars?

Harder... meteors... how do we know those are in our atmosphere and not in the sky?

(Occasionally folks see them actually hit the ground. From the shuttle they can look down on them). No obvious parallax for sun, moon, and stars, (although you can tell parallax for the moon... later..) Planets – wandering stars; comets – hairy stars; meteors – shooting stars.

2. Generally, when you look in the sky you see angular size and angular motion... you don't know how close so how large it is. If someone says a UFO was traveling 500 mph, they really don't know.. they just saw its angular position change across the sky.

3. **Rule of "thumb"**: one thumb at arm's length is about 2 degrees; one fist at arm's length is about 10 degrees; a "hook-em" is 15 degrees. (Generally people with big fists have long arms – check it yourself by measuring "fists" from the horizon to the zenith.

4. **Intensity of energy, light, etc...** is amount of energy (or particles) per unit area.

As the distance from an object gets larger, the surface area of the sphere of radius R enclosing it gets larger $A = 4 \pi R^2$. So the energy PER unit area gets smaller as $1 / R^2$.

5. **Change units**: multiply by "1" to cancel units. $1.609 \text{ km} / 1 \text{ mi} = 1 = 1 \text{ kg} / 2.2 \text{ lb}$, etc.

6. **Flux of Energy** = Energy per unit area PER unit time. 1 W/m^2 is one Joule per second (=1 Watt) crossing an area of one square meter.

Energy from Sun at Earth = $1.4\text{E}3 \text{ W/m}^2$. (actually 1387 W/m^2)

How many kilowatts on average do you use? (KWH used in a month divided by the hours in a month). (Most people around 2 kW, more in summer). How big an area of solar cells do you need? (include day/night; ~10% efficiency; tilt)

7. **Total energy output of Sun** = $1.387\text{E}3 \text{ W/m}^2 * (4 \pi * 1\text{AU} * 1\text{AU}) = 1.74 \text{ E} 4 (1 \text{ AU/m})^2$
 $1.5\text{E}11 \text{ m} = 1 \text{ AU}$ so $1 = 1.5\text{E}11 \text{ m} / \text{AU}$

So Sun's energy output = **solar luminosity** = $1.74 \text{ E} 4 * 1.5 \text{ E} 11 * 1.5\text{E}11 = 3.9 \text{ E} 26 \text{ W}$

8. **components of the galaxy**:

galaxy: a collection of hundreds of *billions* of stars, orbiting around a center of mass.
(some galaxies are smaller)

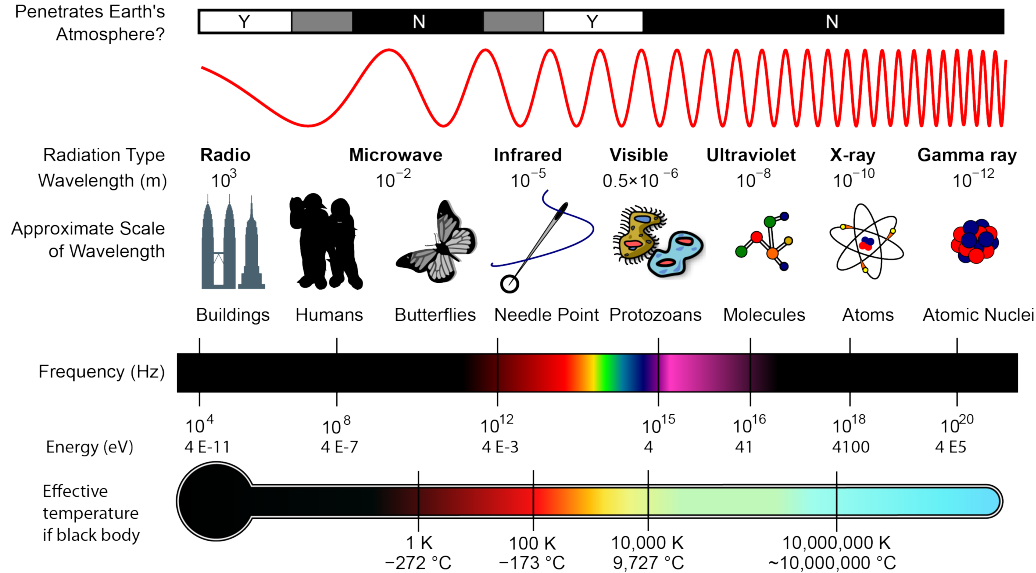
nebulas: cloudy looking balls of gas. Can be birth or death shrouds of stars (some that were called nebulas in the past are actually galaxies on their own – "Island Universes". Thus, the Andromeda nebula is now called Andromeda galaxy).

milky way: the mass of stars in the galaxy our Sun belongs to. A spiral.

stars: hot dense balls of gas, that radiate energy from nuclear fusion

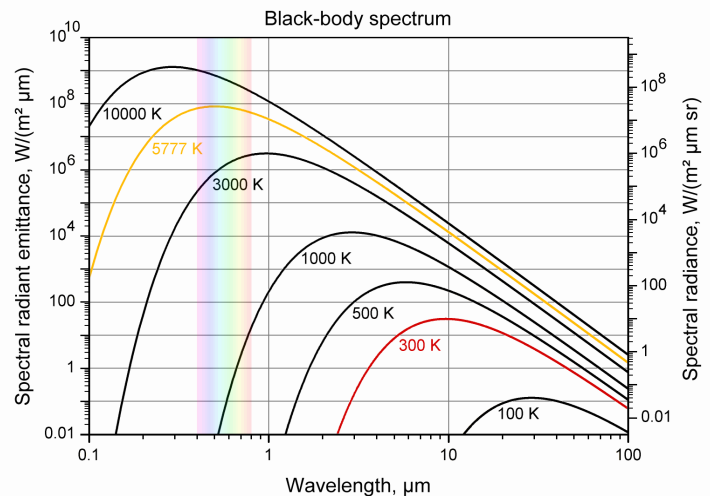
clusters: can be loosely connected ("open clusters") or nearly spherical masses of stars ("Globular clusters").

Light



- Light is an electromagnetic wave**, with properties wavelength λ (m/cyc), speed v (m/s) and frequency f (cyc/sec). Changing magnetic field causes the electric field and a changing electric field causes the magnetic field so it DOES NOT REQUIRE a medium to propagate.
- speed of light:** 3.00×10^8 km/s (186,000 miles per second). The two extra zeros mean that, yes, it is good to three significant digits (actually 2.99792...).
- light slows down when it is in a medium.** So, yes, it is possible that particles can travel faster in a medium than light does *in that same medium* (but not faster than the speed of light in vacuum). If they do, they emit "Cerenkov" radiation (used to detect the particles).
- wavelength of light:** depends on the frequency. $\lambda = c / f$ (watch units!)
(meters / cycle = (meters / s) / (cycles / s))
Unit of measure of **frequency** = Hertz (Hz) = f , cycles per second (contrast radians/s below)
- energy of a photon of light:** $E = h f$ where h = Planck's constant = 6.626×10^{-34} J-s
So, blue and ultraviolet light are more energetic than red or infrared light.
One electron Volt (eV) = 1.602×10^{-19} J. So a photon with frequency f has an energy of
 $(6.626 \times 10^{-34} \text{ J-s} / 1.609 \times 10^{-19} \text{ J}) * f = 4.118 \times 10^{-15} * f$ eV
- angle measured in radians:** the arc length of an angle (the portion of the circumference that the angle subtends) divided by the total circumference ($2 \pi R$). One radian is about 57.3 degrees. A full circle is 2π radians; 90 degrees is $\pi/2$ radians, etc.
- frequency** measured in radians/s (generally lower case greek 'omega') $\omega = 2 \pi f$
So, if a particular color has a frequency 1 MHz, then its frequency in radians/s = 6.3 Mrad/s
 $1 \text{ rad/s} = 2 \pi (\text{rad/osc}) * f (\text{osc/sec})$
- spectrum:** the light from a source spread out so that the intensity at each wavelength can be measured separately.

16. **continuous spectrum:** a spectrum that has light smoothly varying over a large band of wavelengths. (light from a black body or incandescent bulb looks this way) (rainbow)
17. **line emission:** discrete lines of light emission, from a thin gas. Each element has a unique fingerprint of line emissions, from steps in their electron energy levels. (no electrons, no lines, so ionized Hydrogen is invisible). Molecules have broader spectral bands.
18. **absorption spectrum:** a continuous spectrum with narrow dark lines (light from a black body passing through a thin gas)
19. **blackbody spectrum:** a continuous spectrum from a heated solid (an empty box makes a good blackbody too!). Has a peak energy and falls off steeply at high frequency (short wavelength) and slowly at low frequency (long wavelength).



20. **Wien's law:** shows how the wavelength of the peak of a blackbody spectrum changes with temperature. Hotter stars are bluer.
 $\lambda = .3 \text{ cm} / (T/1\text{K})$ where T is measured in Kelvins (from absolute zero)

21. **Stefan-Boltzmann law:** shows how the total energy flux (energy radiated per m^2 per sec) of a black body changes with temperature:

$$JE = \sigma * T^4 \quad (\text{the fourth power of the temperature})$$

Where σ (sigma) is the Stefan-Boltzmann constant = $5.67 \text{ E-}8 \text{ J/(s * m}^2 \text{ K}^4)$

22. **More massive stars are bluer.** Why? More mass means more compression in the core, more nuclear fusion since can overcome the electrostatic repulsion of the protons... Gives out more energy so the surface becomes hotter until the radiation balances the heat flux.
23. **Know the electromagnetic spectrum in order of energy:** radio then infrared then visible (red to blue) then ultraviolet then X-ray then gamma ray. The higher the energy, the more damage it can do. Gamma rays can disrupt nuclei; x-rays can ionize atoms; UV can dissociate atoms and ionize outlying electrons; visible can break molecular bonds (photosynthesis); infrared can only heat. Microwaves heat food by making water or fat molecules vibrate (Cel phones can cook your hands or pop popcorn).
24. **Doppler shift:** if an object emitting sound or light is traveling with the respect to the receiver, the apparent frequency and wavelength will change based on the line of sight velocity V and the speed of the wave C. (C_s = speed of sound; C = speed of light).
 $\Delta f / f = (f - f_0) / f = -\Delta \lambda / \lambda = (\lambda_0 - \lambda) / \lambda = V/C$
Frequency INCREASES if the V is towards you but wavelength DECREASES
Relativistic version: $(\Delta \lambda) / \lambda = -1 + \text{SQRT} ((1 + v/c)/(1 - v/c))$.

Angles, math, etc.

1. **Astronomical Unit** = 1 AU = 1.5×10^8 km = 1.5×10^{11} m which is about 10,000 Earth diameters more accurately is $1.496 \times 10^8 / (2 * 6.378 \times 10^3) = 11727$ Earth diameters
2. **Diameter of Sun** = 1.39×10^9 m which is about 1/100 of IAU distance and about 100 Earth diameters - more accurately is $1.39 \times 10^9 / (2 * 6.378 \times 10^3) = 109$.
3. **Circumference of the Earth:** almost exactly 40,000 km. Approximately 24,900 miles. One mile ~ 1609 meters so you can always multiply by “1” (1.6 km / 1 mi) to change units. ALWAYS ALWAYS SHOW UNITS!!! **Watch for significant figures!!**

Radius (equatorial) of the Earth: 6378 km. Approximately 4000 miles.

4. **Latitude:** angle from equator (north pole is +90, equator is 0, south pole is -90)
North star: is at an angle above your horizon approximately equal to your latitude. (exact is the North Celestial Pole)
Longitude: angle from Greenwich England, positive to the EAST.
5. **Corotation speed at the equator:** 24,900 miles / 24 hours = approx 1000 MPH.
6. **sine of an angle:** Consider a right triangle (one perpendicular angle). The sine of one of the acute angles is the length of the opposite leg divided by the hypotenuse. The cosine of one of the acute angles is the length of the adjacent leg divided by the hypotenuse. Since the two acute angles must add up to 90 degrees (since the sum of the angle of any triangles is 180 degrees), then

$$\sin(\alpha) = \cos(90 - \alpha) = \text{opposite} / \text{hypotenuse}$$

$$\cos(\alpha) = \sin(90 - \alpha) = \text{adjacent} / \text{hypotenuse}$$

$$\tan(\alpha) = \sin(\alpha) / \cos(\alpha) = \text{opposite} / \text{adjacent}$$

Special cases: (MEMORIZE or be able to derive)

A. **45 degree triangle:** both sides are the same, so the $\sin = \text{opp} / \text{hyp} = \text{adj} / \text{hyp}$. From the Pythagorean theorem, $\text{opp}^2 + \text{adj}^2 = \text{hyp}^2 = 2 * \text{opp}^2$, so $\sin(45 \text{ deg}) = \cos(45 \text{ deg}) = \sqrt{1/2} \sim 0.7071$

B. **30-60-90 triangle:** the $\sin(30 \text{ deg}) = 0.5$ (exactly). So $\text{opp} = \text{hyp} / 2$. So the adjacent arm is given by Pythagoras as $\text{opp}^2 = \text{hyp}^2 - \text{adj}^2 = \text{hyp}^2 - (\text{hyp}^2)/4$. Thus the cosine of 30 (= sin of 60) = $\sqrt{3/4} = 0.866$

C. **small angles:** for small angles, the sine of the angle is approximately the same as the size of the angle IF MEASURED IN RADIANS (see below). So for a ten degree angle, the angle in radians is about $10/57.3 \sim 0.1745$. The sin of 10 degrees is actually 0.1736. The smaller the angle, the more accurate this is.

Google calculator: If you don't have a trig calculator, you can type into google “what is $\sin(10 \text{ degrees})$?” CAREFUL: If you ask “what is $\sin(10)$?” you will get the sin of 10 RADIANS. (try asking it a lot of things... “what is the mass of Mars”? kinda cool)

8. **Corotation speed at locations NOT at the equator:** The distance traveled in one spin (day) is the circle of the parallel of latitude. The radius of that circle is $R \cos(\lambda)$, where λ is the latitude. So the corotation speed is at a latitude λ is given by $(40,000 \text{ km} / 24 \text{ hr}) * \cos(\lambda)$
9. **Parallax:** angular offset from viewing from two directions. Only "real" way to measure distance remotely (besides bouncing lasers...)
10. **Parallax angle (radians) = d / D** where d is the spacing of the observers and D is the distance.
 Distance of one **parsec** has a parallax of one arc sec as seen from 1 AU baseline.
 One parsec = $3.262 \text{ LY} = 3.09 \text{ E13 km} = 2.06 \text{ E5 AU}$.
 Farthest distance we **really** can measure = Hubble ~ 3 milliarc sec $\sim 300 \text{ PC} \sim 1000 \text{ LY}$
 Has been expanded by Hipparcos and now by Gaia to ~ 20 micro arc sec!
 (other distances we infer from standard candles)
- 10A **Logarithm of a number y is the value X so that $y = 10^X$.** $\log(X)$ and 10^X are inverse functions. $\log(10^X) = X = 10(\log 10^X)$. To multiply numbers, ADD logarithms.
So $10^2 \times 10^3 = 10^5$ (not 10^6). $\log(4 \times 10^7) = \log(4) + \log(10^7) = 0.6 + 7 = 7.6$
11. **Magnitudes, m :** (apparent magnitude) logarithmic; 0 = Vega; 5 magnitudes is a factor of 100 in brightness. Venus max is about -4; full moon -12.74; Sun is $m_{\odot} = -26.74$

Ratio of brightnesses is DIFFERENCE of magnitudes

$$b_1/b_2 = 10^{((m_2-m_1)/2.5)}$$

$$\log(b_1/b_2) = (m_2 - m_1)/2.5$$

$$m_2 - m_1 = 2.5 \log(b_1/b_2)$$

12. **Absolute Magnitude, M :** Magnitude it would appear if you move the star to a distance of 10 pc. Measures intrinsic luminosity. $M = 4.83 - 2.5 \log(L/L_{\text{SUN}})$ (so yeah, $M_{\odot} = 4.83$)
13. **Distance Modulus: $m - M$:** a measure of the distance to an object.
 $m - M = 5 \log(r / 10 \text{ pc}) - 5 = 5 \log(r / 10 \text{ pc})$
 The farther away something is, the dimmer it is (m gets larger). If $m > M$, it is farther away than 10 pc. If $m < M$, it is closer than 10 pc.

Let's derive the distance modulus.

What is the brightness at distance r compared to the brightness at 10 pc?

Brightness goes as one over the distance SQUARED.

So the brightness b_1 at distance of 10 pc = $L / (10\text{pc})^2$ where L is the intrinsic luminosity of the star.

Similarly the brightness b_2 at distance of r = L / r^2

So what is the ratio of brightness at 10 pc compared to r ? = $(L / (10\text{pc})^2) / (L / r^2) = (r/10 \text{ pc})^2$

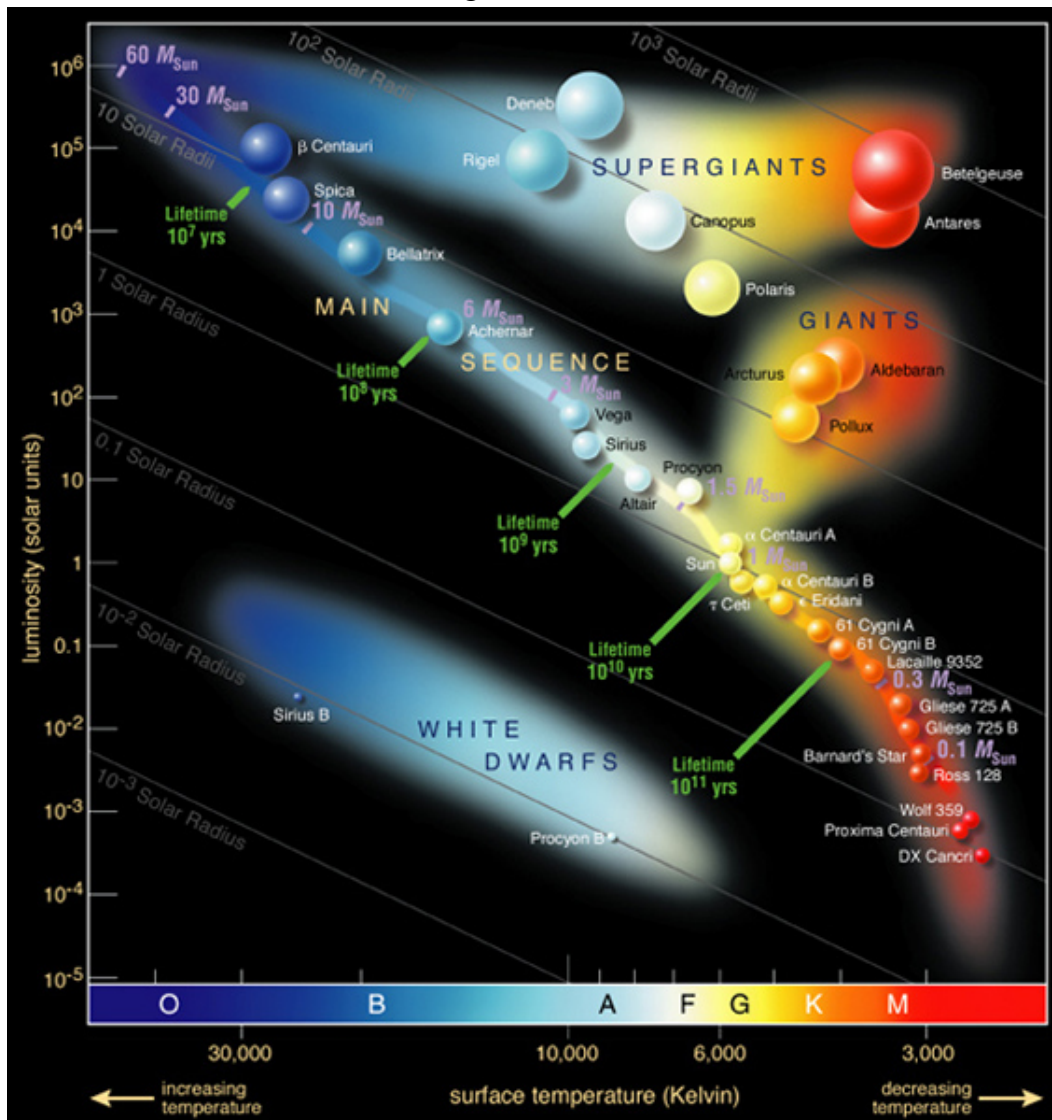
(note the LUMINOSITY DROPS OUT; it's only a function of distance!)

And what is that ratio if expressed in magnitudes?

$\log (b_1/b_2) = \log (r/10\text{pc})^2 = (m_2 - m_1)/2.5$
 here b_2 is at r so $m_2 = m =$ apparent magnitude
 here b_1 is at 10pc so $m_1 = M =$ absolute magnitude

So now our equation is $2 \log (r/10\text{pc}) = (m - M)/2.5$
 $5 \log (r/10\text{pc}) = m - M = 5 \log r - 5 \log 10 = 5 (\log r - 1)$

14. **Standard Candles:** find an object whose luminosity is known (so you know M), measure its apparent brightness m , and calculate its distance from the distance modulus $m - M$. Farther away something is, need brighter and brighter standard candles.
15. **H-R Diagram:** (Herzsprung-Russell) the brightness of a star against its color temperature. Hotter stars are intrinsically brighter. Can get the distance of a cluster by calculating the distance modulus of its HR diagram. (slide the apparent magnitude/temp plot of a cluster up and down till it matches the HR diagram. The difference is the distance modulus, and from that get



the distance).

16. **Main Sequence:** the line on the HR diagram for stars burning Hydrogen (in their main lifetime). **When on the main sequence, the gravitational force from higher levels pushing down is balanced by the thermal pressure nkT of the inner portions created by the fusion in the core. This is called **hydrostatic equilibrium**. This kind of balance of forces occurs in every gaseous body, including planets and our own atmosphere.**

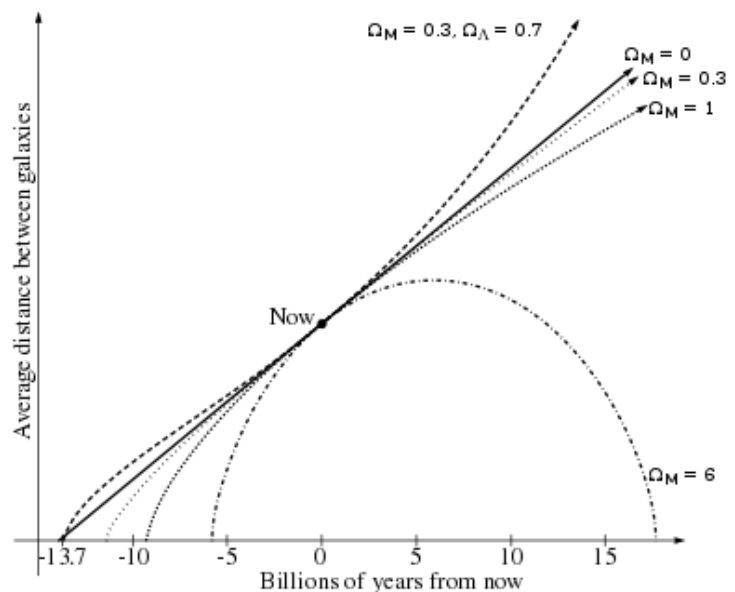
Very small stars don't reach nuclear fusion at all, radiating only from gravitational contraction (brown dwarfs). Needs a minimum of 75 Jupiter's mass to have fusion. Those slightly bigger, but still small mass stars (less than 0.8 solar masses) fade away when they use up their Hydrogen, turning into red dwarfs.

Spectral class: Hottest stars on main sequence are "O" then "B" then F G K M "Oh, be a fine girl, kiss me!". The Sun is a G class star.

(yeah, there is a jump from 16 to 25. I moved some stuff around and didn't renumber).

25. **Hubble Constant:** Galaxies farther away are moving faster from us. No real "center" of the universe (looks the same to all). **$H = 71 \text{ km/s / Mpc}$ (68)** (so a Galaxy 1 Mpc away is typically moving 71 km/s away from us; if it is 3 Mpc away, it is moving $3 \text{ Mpc} \times 71 \text{ (km/s Mpc)} = 213 \text{ km/s}$ away from us).

26. **Age of the Universe:** $= 1/H$ if the universe is expanding at a constant rate. $= 13.8 \text{ BY bp}$. Zero time = "**Big Bang**" (universe all together and very small and hot, then particles condensed, then stars, galaxies, etc).



-----The following is from the second half of the course (not covered in Quiz 1) -----

27. **Dark Matter** = real matter which is hard to see (might be brown dwarfs, black dwarfs, black holes, etc..) (about 27% of the total energy in the Universe, or about 85% if you exclude dark energy). (Inferred from the rotation of galaxies – outer parts of galaxies rotate faster than they should if all the mass were visible). "Regular" matter around 5%. Appears to be associated with normal matter. Most likely = WIMPS (Weakly Interactive Massive Particles, like Higgs bosons or other strange stuff), or MACHOs (MASSive Compact Halo Objects), like black holes, neutron stars, etc. Best estimate is MACHOS can be about 10% but the rest is probably WIMPS.
28. **Dark Energy** = energy that is causing the expansion of the Universe to accelerate. About 68% of the energy in the Universe (but is constantly INCREASING). Strange!! (and might even go away as we better measure the distances to very far away galaxies).
Good website: <http://science.nasa.gov/astrophysics/focus-areas/what-is-dark-energy/>
29. **atomic number**: number of protons in the nucleus (Z). 1 = H, 2 = He, 3 = Li, etc.

Group→	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
↓Period																		
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo
				* 57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
				** 89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

30. **atomic weight**: number of protons plus neutrons in the nucleus (=A). For large nuclei, there are roughly twice as many neutrons as protons; for smaller nuclei, the ratio is more one-to-one. Notation: ^3He means Helium with 3 total weight. Since Helium implies two protons, then only one neutron. Typically shown as a superscript to the left of the atomic symbol (e.g., ^{12}C). Carbon 12 has $Z = 6$ and $A = 12$ so 6 neutrons. Carbon 14 (^{14}C) has $A=14$ and $Z=6$ so has 8 neutrons and is radioactive.
31. **isotope**: two forms of the same element (same Z), but with different numbers of neutrons, so different A's. Example: Carbon 14 (^{14}C) versus Carbon 12 (^{12}C). Often one or two isotopes will be stable, and the rest radioactive.

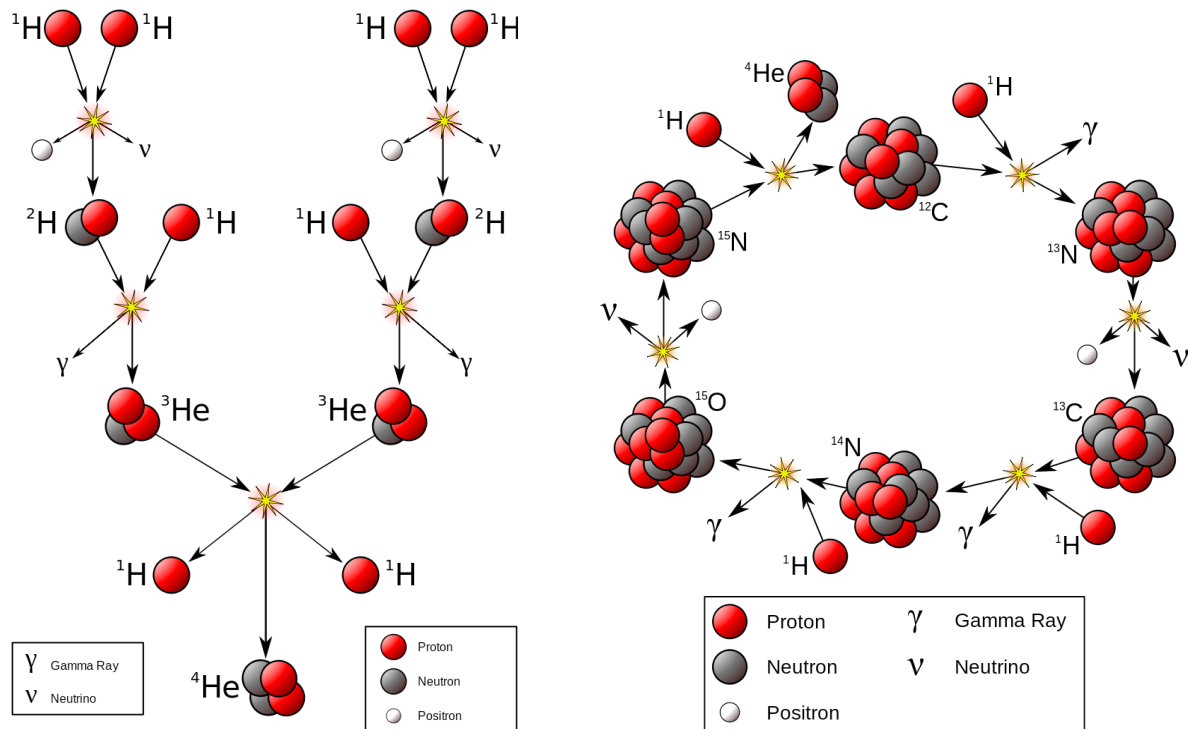
32. **nuclear fusion:** merging of two nuclei to create a new nucleus of a different element, releasing energy, plus subatomic particles like beta particles, gamma rays, neutrinos.
 Example: $4\text{H} \rightarrow \text{He} + 2\text{ positrons, plus 2 neutrinos, gamma rays, and 24.7 MeV energy.}$
 You can get the energy released in fusion by measuring the difference in mass between the total initial mass (e.g. four protons) and the final mass (e.g. one Helium nucleus). Ends up to be 0.7% (.007). If the mass goes UP, you LOSE energy.

alpha particle: Helium nucleus (two Hydrogen, two Neutrons)

beta particle: electron

positron: antiparticle for electron; has electron mass but positive charge.

neutrino: neutral particle with momentum and energy but no rest mass (!)
 inferred from conservation of mass and momentum.



Proton-proton fusion

CNO cycle (Carbon is reused)

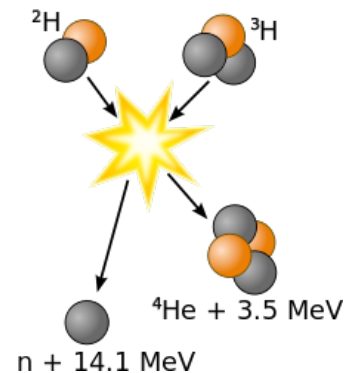
Easier: Deuterium / Tritium : ${}^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + n + 17.6\text{ MeV}$

Or, even better, ${}^3\text{He}$ fusion: ${}^2\text{H} + {}^3\text{He} \rightarrow {}^4\text{He} + p + 18.3\text{ MeV}$

or ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2p + 12.86\text{ MeV}$

Big advantage of He^3 - no emitted neutrons so container doesn't become radioactive.

CNO cycle – the Carbon is reused (not used up), and it is a bit easier to do, but requires a second-generation star (like our Sun)



33. **light** has both wave (e.g. diffraction, interference) and particle (energy, momentum) properties. A particle of light is called a **photon**. (A particle of sound energy is a phonon).
gamma particle (ray): a very high energy photon, capable of influencing the nucleus.
34. **nuclear fission**: splitting of a large unstable nucleus into two or more pieces. Changes the single element with initial Z and A to two or more elements each with smaller Z and A.
35. **radioactive decay**: the change of one atom to another, by the emission of one or more subatomic particles (alpha, beta, gamma, or fission). So, beta decay: radioactive decay by emitting an electron. Changes the charge of the nucleus by +1 (changes a neutron to a proton).
36. **half-life**: the time for half of the remaining radioactive material in a sample to decay. Short half-life = very radioactive.
37. **rest energy**: the energy of a particle when at rest. Light and probably neutrinos have zero rest energy.
38. **strong force**: very strong, but very short-range force that keeps the nucleus together, overcoming the extremely strong electrostatic repulsion. So fusion only occurs deep in stars, where the heat and pressure is so high that particles can bump together, despite the repulsion. (that's why it's so hard to do it on earth!)
39. **stellar evolution**: Changes in appearance, density, size, brightness, etc of a star during its lifetime. Generally plotted on a Hertzsprung-Russell diagram. End products of stars depends on whether it was a small star, a big star, or a VERY big star.
40. **age of a cluster**: When you plot the HR diagram of a cluster (using m instead of M), the typical age of the hottest star still on the Main Sequence tells you the age of the cluster. Anything hotter has already evolved off the Main Sequence so the cluster is at least that age).
41. **death of a sunlike star**:
 Hydrogen burning runs out in core, core collapses, Helium starts to burn, expanding the envelope. Turns first to yellow then red giant, then core turns into **white dwarf**. White dwarfs are held up by electron degeneracy pressure (no more than two electrons can share an electron orbit). Electron degeneracy limit is 1.4 solar masses (M_{SUN}) (the Chandrasekhar limit). So long as the white dwarf is less than 1.4 M_{SUN} , the electrons stay separate from the nucleus. The Sun will become a white dwarf. When its carbon burns out it will be a black dwarf. If a white dwarf accretes more than 1.4 M_{SUN} , it can compress even more (the electrons get shoved into the nucleus, becoming a neutron star and a "type 1" supernova.) If the star is just a bit heavier than the Sun, the outer shell becomes a **Planetary nebula - gorgeous wispy colorful outer shell of a dying star, e.g. eskimo or ring nebula (contrast to supernova remnant, which is the outer remnant of the supernova shock wave, like the crab nebula)**.
 Here is a Hubble video of a star ejecting a shell over four years: https://gizmodo.com/star-explosion-video-is-the-most-awesome-thing-i-have-e-1589138376?fbclid=IwAR0VYqAe-MjMbZ1HQ-Fc9glI2RBQbJAehlCbjHH6htEa410xV_TxDN-if8E

42. **death of a heavy star:** even heavier mass stars (greater than $8 M_{\text{SUN}}$) become Red Supergiants and can burn progressively more massive ash in progressive layers in their core (e.g. He to Carbon, Carbon to Oxygen, etc). Iron is the most stable element... so creating elements heavier than iron takes energy instead of giving it off, causing the core to collapse. The outer layers (which were lighter than Iron) falling inward then fuse explosively, creating a

43. **supernova** – a blast when a star can appear as bright as an entire galaxy. The shockwave is what expels the heavier materials (heavier than Iron) and peppers the galaxy with heavy materials. The expanding shock wave eventually dissipates, becoming a **supernova remnant**. Since the Sun has heavy materials in it, it must be a "second generation" star, with material in it from a previous supernova. The core remnant, now bigger than 1.4 solar masses, becomes a

44. **neutron star**, with all electrons and protons squeezed into neutrons. M_{SUN} material known – 10^{18} kg/m^3 . So, if it has a mass of $1.4 M_{\text{SUN}}$, how large (radius) will it be?

$$\rho = 1.4 M_{\text{SUN}} / (4/3 \pi R^3) = 10^{18} \text{ kg/m}^3$$

$$\text{So } R^3 = 1.4 M_{\text{SUN}} / (4/3 \pi \times 10^{18} \text{ kg/m}^3) = (4.2 \text{ E } 30 / 4.19 \text{ E } 18) \text{ m}^3 = 1.0 \text{ E } 12 \text{ m}^3$$

$$\text{So } R = \text{E}4 \text{ m} = 10 \text{ km.}$$

45. **pulsar:** a special kind of neutron star that emits pulses of radio waves at its spin frequency, which can start out to be milliseconds but gradually slows down. Pulsars are calculated to have enormous magnetic fields. The Crab pulsar (the supernova of 1054) pulses in all wavelengths – optical, radio, and even X-ray and gamma ray!

Here is great web page of pulsar sounds: (radio waves played through audio equipment):

<http://www.jb.man.ac.uk/research/pulsar/Education/Sounds/>

46. **Death of a superheavy star** ($>25 M_{\text{SUN}}$). Similar to heavy in that it becomes a Red Supergiant and supernova. The remaining core remnant is so heavy ($> 3 M_{\text{SUN}}$) that it cannot fight off gravity and becomes a black hole.

47. **stellar black hole** – with mass greater than $3 M_{\text{SUN}}$, the neutron degeneracy pressure is not enough to hold up the star's core, and core falls in on itself and becomes a stellar black hole.

48. **Schwarzschild radius** – Maximum size of a black hole for a given mass. The escape velocity becomes the speed of light so light cannot escape. (It can orbit at a slightly greater distance). For a one solar mass object, the Schwarzschild radius is only 3 km!

escape velocity v from a star or planet with mass M and radius R $v = \text{Sqrt}(2 GM / R)$.

So, the radius where the escape velocity becomes c is

$$R_{\text{sch}} = 2GM / c^2$$

When something enters the Schwarzschild radius, we can't get any information back from it, so it is also called the **event horizon**.

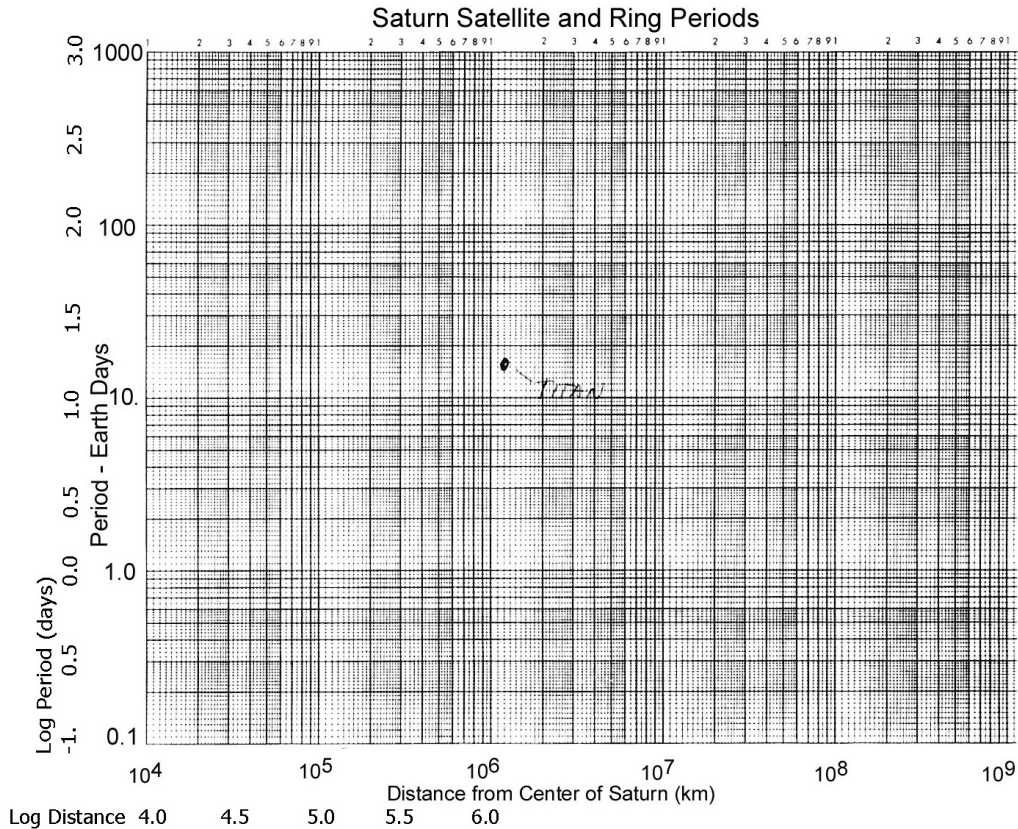
49. **supermassive black holes:** like the 4.3 Million solar mass black hole at the center of the Milky Way, probably caused by collisions of many black holes absorbing each other. Near collisions can eject black holes into the galactic halo, becoming part of the MACHO objects. We now know the mass of the black hole (called "Sagittarius A") by watching other stars orbiting (see video here: <https://www.eso.org/public/videos/eso1825e/>)
50. **Exoplanets:** planets discovered around other stars. **Can be discovered measuring the star's wobble ("astrometry"), either by Doppler shift or direct observation, or by measuring small changes in star brightness as a planet transits the star "transit" or "astrophotometry".** Which works best if the plane of the distant solar system is in the plane of the sky? (position) Which works better if the plane of the distant solar system is perpendicular to that plane? (Doppler). Total confirmed so far: 5000(!), mostly by transits.
51. **Transit method of finding exoplanets:** Know how measuring the light output from a star very accurately can determine the presence of planets around stars. What special thing must be true about the orbit of the planet around the star as seen from Earth? (exactly in plane of the orbit, **so that the planet transits the star once per orbit**). Which spacecraft is measuring these? (Kepler began but **is finished now, but a new mission TESS is in operation**) What do we call this process? (eclipses or occultations or transits, part of spectrometry).
52. **Direct Imaging:** It is difficult to separate a planet from the star to make an image of it. Nevertheless, Hubble has done it recently, and also Gemini, using infrared light (<http://blogs.discovermagazine.com/badastronomy/2008/11/13/huge-exoplanet-news-items-pictures/>).
53. **Microlensing:** 89 exoplanets (so far) have been discovered by microlensing, <https://exoplanetarchive.ipac.caltech.edu/cgi-bin/TblView/nph-tblView?app=ExoTbls&config=microlensing&constraint=mlmodeldef=1>
When a small planet passes directly in front of a distant star, for a few moments you can see multiple images of the distant star.
<https://www.planetary.org/explore/space-topics/exoplanets/microlensing.html> and <https://www.universetoday.com/138141/gravitational-microlensing-method/>

Exoplanet archives: NASA exoplanet archive: <https://exoplanetarchive.ipac.caltech.edu>
ESA archive: <http://exoplanet.eu> (can do plots e.g. "hot jupiters")
Older archive: <http://exoplanets.org>
54. **James Webb telescope:** Launched Dec 25, 2021. Infrared sensitivity using a multiple mirror honeycomb design. Uses sunshades to keep sensors cool. Stationed at Lagrangian point L2 so doesn't get light or heat reflected off the Earth. (faces AWAY from the Sun and away from the Earth and moon.)

55. **Solar flare:** sudden dramatic brightening of a small region of the sun. Sunspots and solar flares more common at the peak of the **sunspot cycle**, an 11-year cycle of solar variability. At solar max, the sun's magnetic field flips, making a 22-year magnetic cycle. Solar flares often release very energetic particles that travel near the speed of light (Solar Energetic Particles - SEP's) and can release a
56. **coronal mass ejection (CME)**, a blast of solar wind from an erupting filament, which is **much faster and denser** than usual which reaches earth in about two days. A CME can cause a geomagnetic storm, with auroral and energetic particles which can harm astronauts and spacecraft.

More stuff you should already know:

57. **Right ascension:** stellar coordinate similar to longitude, but often measured in HOURS. Zero at the position of the Sun at the vernal equinox.
- Sidereal Time:** Right ascension of the stars on your meridian (line going from north star to southern horizon).
58. **Declination:** stellar coordinate similar to latitude. Zero at equator; +90 deg at North Celestial Pole.
59. **Precession:** movement of the Celestial Pole (and therefore also the vernal equinox) because of the changing direction of the Earth's spin axis, caused by Moon tides. Goes around once in 23,000 years. Star charts use "epochs" (1950, 2000, etc.) since BOTH coordinates change slightly.
60. **Solar day:** 24 hours. Noon to noon (sun aligned).
61. **Sidereal day:** 24 hours – 4 minutes. Star overhead to the same star overhead again, which is the TRUE rotation period. Shorter than 24 hours because of the Earth's revolution around the Sun.
62. **Logarithm:** if you write a number as a power of ten, the logarithm is that power, which can be a fractional number. Examples: $\log(1000) = \log(10^3) = 3$. $\log(\sqrt{10}) = \log(10^{.5}) = .5$ And since $\sqrt{10}$ is approximately π , $\log(\pi) = .5$ and $\log(\sqrt{\pi}) = 0.25$.
- general rule: $\log(10^{**x}) = x$
63. Multiplying numbers, you add the logarithms; dividing numbers, you subtract the logarithms. This is how a slide rule works, marked out as logarithms.
- $\log(a * b) = \log(a) + \log(b)$
 $\log(a/b) = \log(a) - \log(b)$
64. Logarithm of powers = multiply the logs
- $\log(a ** b) = b \log(a)$
 $\log(\text{bth root of } a) = (1/b) * (\log(a))$



65. **log-log graph paper:** plots of the log of one number versus the log of the other.

So, it is great for plotting relationships that are POWER LAWS of one another, like Kepler's third law:

$$T^2 = k a^3 \quad \text{so, taking the log}$$

$$2 \log T = \log(k) + 3 \log(a)$$

$$\log T = 1/2 \log k + 3/2 \log a \quad (\text{dividing by 2})$$

so if $y = \log T$ and $x = \log a$, you get a line with slope $3/2$ and intercept $(\log k)/2$

so, for the log-log plot of satellite periods and distances, the intercept is $(\log k)/2$

and for Kepler's law, the kepler k is given by $(2 \pi)^2 / G M$

so the ratio of two planets' Masses can be calculated from the offsets of the two lines.

$$\text{Intercept 1} = \log(k_1) / 2$$

$$\text{Intercept 2} = \log(k_2) / 2$$

$$\begin{aligned} \text{Intercept 1} - \text{intercept 2} &= (1/2) [(\log ((2 \pi)^2 / G M_1)) - (\log ((2 \pi)^2 / G M_2))] \\ &= (1/2) \log (M_2 / M_1) \end{aligned}$$

So, you take the offset of the two lines (in the log, by seeing where the lower trace crosses an even power of ten and then taking the value of the upper), and squaring it, to get the value of the ratio of the masses.

66. **exponential e** is another "irrational" number, approximately 2.718....

$$e = 1 + 1/2 + 1/3 + 1/4 + 1/5 \dots$$

“exponentially increasing” function (like the amount of yeast in bread dough, Covid 19 cases, numbers of humans on earth..)

$$n = N_0 * e^{t/T} \quad \text{where } T \text{ is a “time constant”, the time to increase by a factor of } e$$

Similarly, “exponentially decreasing” function (like half-life of radioactive material) is

$$n = N_0 * e^{-t/T} \quad \text{where } T \text{ is a time constant and } N_0 \text{ is the starting density}$$

67. Earth’s atmosphere density n is exponentially decreasing with height h , so its function is

$$n = N_0 * e^{-h/H} \quad \text{where } N_0 \text{ is the surface density and } H \text{ the “scale height”}.$$

68. **Natural logs** = log base e (as opposed to log base 10) = $\ln(x)$

$$\ln(e^{**}y) = y$$

69. relationship between \ln and \log is easy!

$$\text{Log}(e^{**}y) = y \log e = y * (0.434\dots) \quad [\log(e) = 0.434\dots]$$

$$\text{Ln}(e^{**}y) = y$$

$$\text{So, for any number } y, \text{ Log}(y) = 0.434 \ln(y)$$

$$\text{Ln}(10^{**}z) = z \ln(10) = 2.3 * z \quad [\ln(10) = 2.302\dots = 1/\log(e)!]$$

70. **semi-log paper** is log on one axis (generally the y axis) and linear on the other (generally x).

When you do that, then exponential functions plot as straight lines.

$$n = N_0 e^{-h/H}$$

$$\text{Log } n = \log(N_0) - (\log(e^{-h/H}))$$

$$\text{Log } n = \log(N_0) - (h/H) \log e$$

$$\text{So, if you plot } y = \text{Log } n \text{ and } x = h$$

Then you get a straight line with intercept of $(\log(N_0))$ and slope $(0.434 / H)$.

The atmospheric density (or amount of radioactive material) gets smaller and smaller with height, but never really goes to zero, but it does get so small that it is hard to measure.